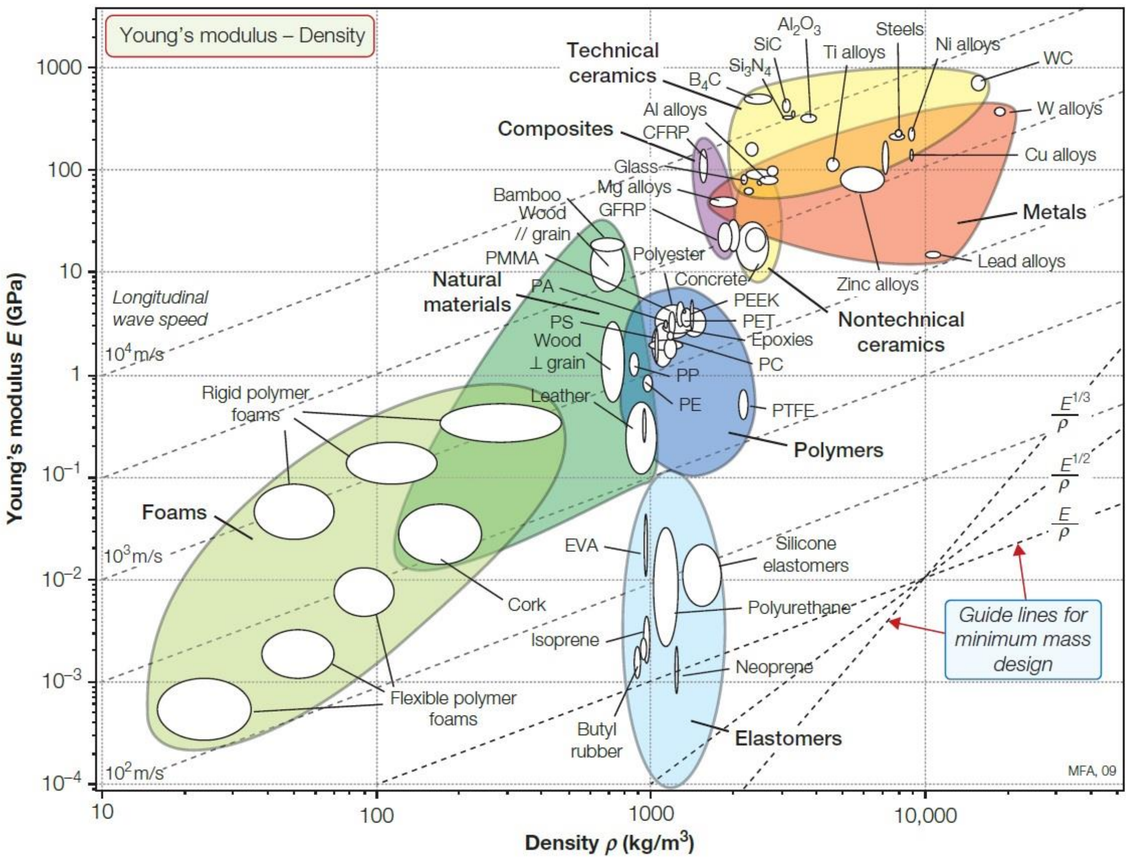
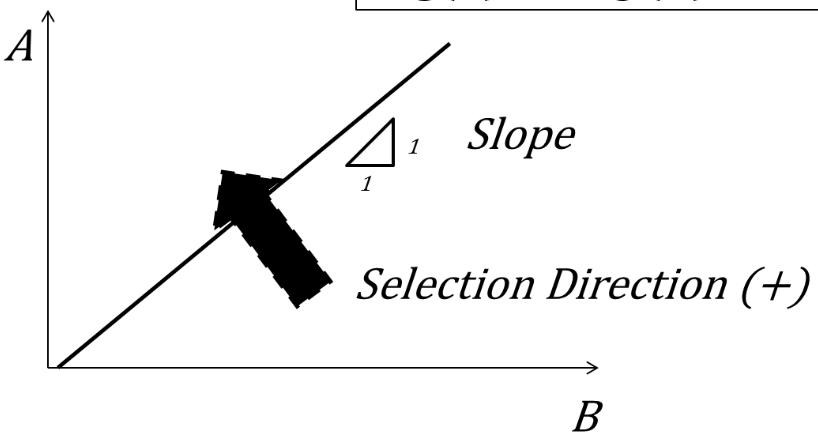
Ashby diagrams



Arrange the index

$$I = \frac{A}{B} \longrightarrow Log(I) = Log\,(A) - Log(B)$$

$$Log(A) = Log(B) + Log\,(I)$$



Resistance and stiffness in TRACTION and FLEXION mode

To Minimize the Mass

$$m = A \cdot L \cdot \rho$$

$$A = \frac{m}{L \cdot \rho}$$

Where m= mass
A = section area
L = length
ρ = density

Understand the goal → Stiffness must be lower down limited (TO MASSIMIZE)
Resistance must superiorly be limited (TO HOLD UNDER A LIMIT)

To Minimize the Cost

If the material price is Cm $/kg, the cost of the material to make a component of mass m is just mC_m.
The objective function for the material cost C of the piece then becomes

$$C = m\, C_m$$

Case : Beam under traction conditions

Stiffness Optimization

$$\frac{F}{\delta} \geq S_{min} = S$$

$$\frac{E \cdot A}{L} \geq S \rightarrow \frac{E \cdot m}{L^2 \cdot \rho} \geq S \rightarrow m \geq S \cdot L^2 \cdot \frac{\rho}{E}$$

$$M = \frac{E}{\rho}$$

Resistance Optimization

$$\frac{F}{A} \leq \sigma_f$$

$$\frac{F}{m/(L \cdot \rho)} \leq \sigma_f \rightarrow m \geq F \cdot L \cdot \frac{\rho}{\sigma_f}$$

$$M = \frac{\sigma_f}{\rho}$$

Case : Beam under flexion conditions

Stiffness Optimization

$$\frac{F}{\delta} \geq S_{min} = S$$

$$C_1 \cdot \frac{E \cdot J}{L^3} \geq S \text{, with } J = \frac{b \cdot w^3}{12} \quad \text{[Moment of inertia, b width , w thickness]}$$

- For square beam $b=w,\ A=b^2 \rightarrow J = \frac{A^2}{12}$

[this hypothesis is correct also if b and w are different but in proportion, as $b=C \cdot w$]

$$C_1 \cdot \frac{E \cdot A^2}{12 \cdot L^3} \geq S \rightarrow \frac{m^2}{L^2 \cdot \rho^2} \geq \frac{12 \cdot L^3}{C_1 \cdot E} \cdot S \rightarrow m^2 \geq \frac{12 \cdot L^3}{C_1} \cdot S \cdot \frac{L^2 \cdot \rho^2}{E}$$

$$m \geq \left(\frac{12 \cdot S}{C_1 \cdot L} \right)^{\frac{1}{2}} \cdot L^3 \cdot \frac{\rho}{E^{\frac{1}{2}}}$$

- For beam with width fixed and thickness variable (= panel), w=A/b

$$\rightarrow J = \frac{b \cdot \left(\frac{A}{b} \right)^3}{12} = \frac{A^3}{12 \cdot b^2}$$

$$C_1 \cdot \frac{E \cdot A^3}{12 \cdot b^2 \cdot L^3} \geq S \rightarrow \frac{m^3}{L^3 \cdot \rho^3} \geq \frac{12 \cdot L^3 \cdot b^2}{C_1 \cdot E} \cdot S \rightarrow m^3 \geq \frac{12 \cdot L^3 \cdot b^2}{C_1} \cdot S \cdot \frac{L^3 \cdot \rho^3}{E}$$

$$m \geq \left(\frac{12 \cdot S \cdot b^2}{C_1} \right)^{\frac{1}{3}} \cdot L^2 \cdot \frac{\rho}{E^{\frac{1}{3}}}$$

Case : Beam under flexion conditions

Resistance Optimization

$$\sigma_{max} = \frac{M_f \cdot y_{max}}{J} = \frac{M_f \cdot \left(\frac{w}{2}\right)}{J} = \frac{M_f}{J'} \leq \sigma_f$$ [y_{max}= Maximum distance from neutral axis]

with $J = \frac{b \cdot w^3}{12}$ [Moment of inertia, b width , w thickness]

and with $J' = \frac{b \cdot w^2}{6}$ [b width , w thickness]

- For square beam $b=w$, $A=b^2 \rightarrow J' = \frac{A^{3/2}}{6}$

[this hypothesis is correct also if b and w are different but in proportion, as $b=C \cdot w$]

$$\frac{M_f}{J'} = 6\frac{M_f}{A^{3/2}} = 6\frac{M_f}{A^{3/2}} \leq \sigma_f$$

$$\rightarrow 6\frac{M_f}{A^{3/2}} = 6\frac{M_f \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}} \leq \sigma_f$$

$$m^{3/2} \geq 6 \cdot \frac{M_f \cdot L^{3/2} \cdot \rho^{3/2}}{\sigma_f}$$

$$m \geq (6 \cdot M_f)^{2/3} \cdot L \cdot \frac{\rho}{\sigma_f^{2/3}}$$

- For beam with width fixed and thickness variable (= panel), w=A/b

$$\rightarrow J' = \frac{b \cdot w^2}{6} = \frac{b \cdot \left(\frac{A}{b}\right)^2}{6} = \frac{A^2}{6 \cdot b}$$ [b width , w thickness]

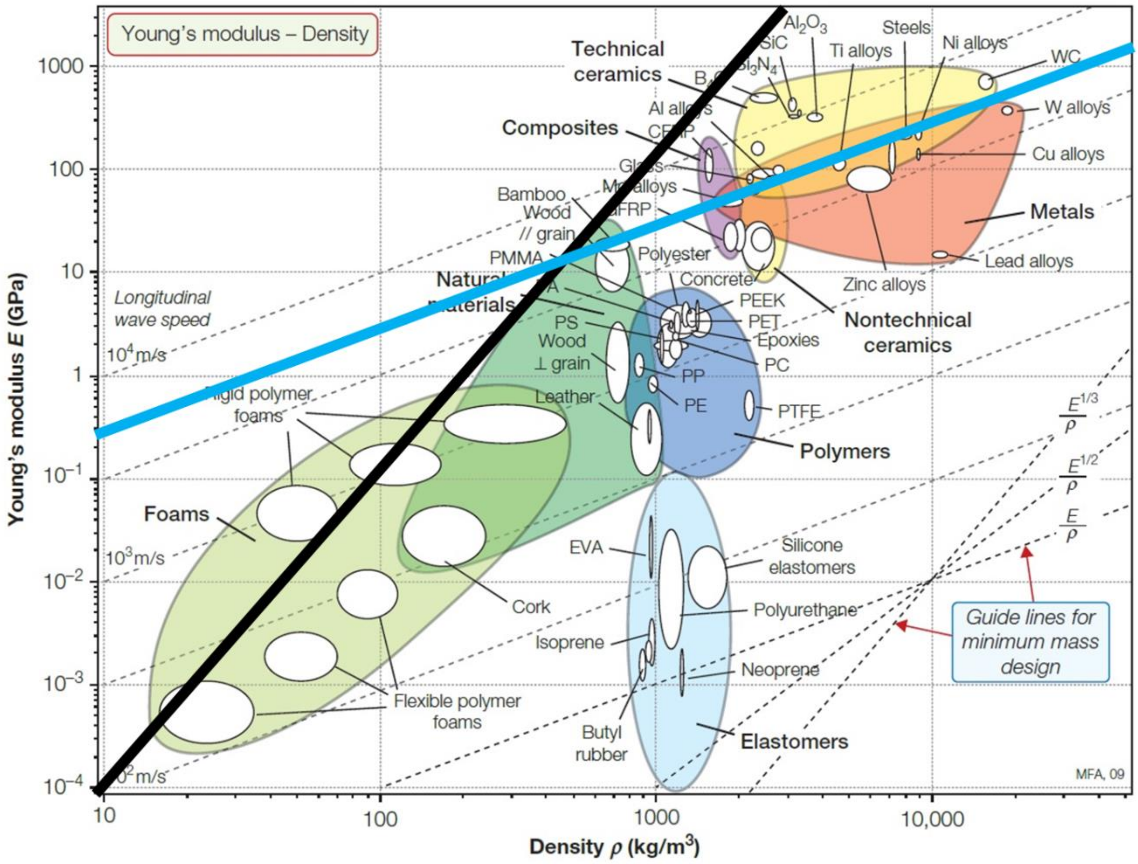
$$\frac{M_f}{J'} = 6 \cdot b \cdot \frac{M_f}{A^2} \leq \sigma_f$$

$$\rightarrow 6 \cdot b \cdot \frac{M_f}{A^2} = 6 \cdot b \cdot \frac{M_f \cdot L^2 \cdot \rho^2}{m^2} \leq \sigma_f$$

$$m^2 \geq 6 \cdot b \cdot \frac{M_f \cdot L^2 \cdot \rho^2}{\sigma_f}$$

$$m \geq (6 \cdot b \cdot M_f)^{1/2} \cdot L \cdot \frac{\rho}{\sigma_f^{1/2}}$$

Important don't confuse one index (one condition) with another index, as instance:



Exercises:

ES0 → simple Resistance Selection

ES1→ Resistance and Stiffness Selection for Sheets

ES1'→ Resistance and Stiffness Selection for Cheap Sheets

ES2→ Selection for Oars light and stiff

ES2'→ Selection for Oars light, stiff and CHEAP

ES3→ Minimize Thermal Distortion for precision devices

ES3→ Minimize Thermal Distortion for precision devices +price consideration

ES4→ Selection for long span transmission line

ES4'→ Selection for conductive and light cable

ES 5→ Materials for springs

PRACTICAL WORKSHOP related exercises

Materials Selection – Academic Year 2016-2017

Prof. J. Lecomte-Beckers

Pick 3 exercises below, with at least 1 amongst the exercise 4 and 5. Solve the problems as performed during the workshop. An analytical resolution is required. If some hypotheses are necessary, you should detail them. The choice of the free variable has to be strictly justified. Some extra limits related to manufacturing, durability or mechanical aspects could have to be implemented.

The formulas datasheet present at the end of the printed course can be used.

The report must be printed and brought to the assistant Maurizi Enrici.

Limit date: 18th November 2016

Exercise 1

Select the cheapest materials for the Parthenon columns with a rectangular cross-section.

Exercise 2

Select the cheapest material for a can.

Exercise 3

Select the lightest and cheapest material for a car hood.

Exercise 4

Select the lightest material for a lightweight table with slender cylindrical legs.

Exercise 5

Select the lightest material for a tube which is submitted to bending and torsion.