Materials Selection – Case Study 1
Bases and Mechanical Properties

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Mechanical Properties Case Studies

- Case Study 1: The Lightest STIFF Beam
- Case Study 2: The Lightest STIFF Tie-Rod
- Case Study 3: The Lightest STIFF Panel
- Case Study 4: Materials for Oars
- Case Study 5: Materials for CHEAP and Slender Oars
- Case Study 6: The Lightest STRONG Tie-Rod
- Case Study 7: The Lightest STRONG Beam
- Case Study 8: The Lightest STRONG Panel
- Case Study 9: Materials for Constructions
- Case Study 10: Materials for Small Springs
- Case Study 11: Materials for Light Springs
- Case Study 12: Materials for Car Body

CES 2009

CES 2016
Materials selection

- **Mechanical properties**: tensile test, fatigue, hardness, toughness, creep...

- **Physical properties**: density, conductivity, coefficient of thermal expansion

- **Chemical properties**: corrosion

- **Microscopic characteristics**: anisotropy of properties, hardening, microstructure, grain size, segregation, inclusions...
Materials selection

- **Process linked aspects**: formability, machinability, weldability, stampability
- **Aesthetic aspects**: colour and surface roughness

Notice: surface properties ≠ volume properties

4 poles for engineering and material science
Design steps

**Design tools**
- Functional analysis
- Methods for fonction analysing
- 3D modeler
- Simulation
- Optimization methods
- Components modelization (FEM)
- DFM/DFA

**Objectives**
- Understand the function
- Define the main characteristics of the product
- Optimize shape
- Optimize realisation (manufacturing + assembling)

**Market Need**

**Materials Selection**
- Choose between main classes of materials (ceramics, metals...)
- Choose between families inside a material classe (steel, cast iron, Al...)
- Choose a particular variety inside the family (6000 or 7000 alloy...)

**Process selection**
- Choose between the main classes of processes (moulding, machining...)
- Choose between the families for a given process class (sand casting, pressure casting...)
- Choose between the different varieties for a given family of process (moulding, metal mould, Cosworth process...)

**Principle**

**Improvement**

**Detailing**
Forecasts

Evolution of materials is challenged by:

- mechanical properties
- physical and chemical properties
- environmental problems (manufacturing)
- materials ressource

Key Domains: energy (nuclear, solar cells, ...) transport
A bit of History
A bit of History

1850s, time of the Crimean War

Napoleon III

French military engineers had found they could control the trajectory applying a rifling or “spinning” in the barrels of guns (cannon)

The spiraling motion added extra stresses

Consequence?

Need a higher-strength material → Steel
A bit of History

1946, University of Pennsylvania Moore School of Electrical Engineering

Electronic Numerical Integrator Analyser and Computer (Eniac) by John Mauchly and J. Presper Eckert

The first general-purpose electronic computer
17468 thermionic valves
70,000 resistors
Covered 167 square metres of floor space
Weighed 30 tonnes
Consumed 160 kW of electricity

1947,
Discovery of the Transistor (Semiconductors)

Built from materials such as Silicon and Germanium which can either behave as an electrical insulator or conductor

Companies spent tens of billion of dollars to squeeze more circuits on to a small ‘chip’ of material

2010, an Intel X3370 microprocessor – 820 million transistors
Your computer could handle 3 billion instructions /s
600000 more than Eniac
A bit of History

2012, low cost airlines company

Change the material of a small pivot (46) for each seat

In the air transports
Weight = Costs

Aluminum $\rightarrow$ PE+ Glass fibers Composite

10,000,000 dollars saved each year
Mike Ashby from University of Cambridge
Ashby Diagrams

[The mechanical efficiency of natural materials, Mike Ashby, 2003]
Ashby Diagrams
Ashby Diagrams
Ashby Diagrams
Ashby Diagrams
Ashby Diagrams

- Dimensionless wear constant $K = k_s H$
- Wear rate – Hardness
  - Metals
  - Medium carbon steels
  - Stainless steels
  - High carbon steels
  - Low alloy steels
  - Tool steels
  - WC
  - Technical ceramics
  - Polymers and elastomers
  - Cast irons
  - Bronze
  - Silica glass
  - $\text{Al}_2\text{O}_3$

- Coefficient of friction on dry steel $\mu$
  - Butyl rubber
  - Natural rubber
  - Lead alloys
  - Borosilicate glass
  - Cu alloys
  - WC
  - Cast irons
  - PA
  - PP
  - PE
  - Wood
  - PTFE

- $\mu$ for boundary lubrication = 0.01 – 0.1
- $\mu$ for full hydrodynamic lubrication = 0.001 – 0.01

- Graphs show the relationship between wear rate and hardness for different materials.

Thursday, October 4, 2018  Materials Selection  18
Simplification: Where is the problem?

For a beam under flexion, the moment of inertia: \( I_{XX} = \frac{bh^3}{12} \)

Length (L): 300 mm \( I_{XX} = \frac{25 \cdot 1^3}{12} = 2,1\ mm^4 \)

Thickness (h) = 1 mm \( I_{YY} = \frac{1 \cdot 25^3}{12} = 1300\ mm^4 \)

Width (b) = 25 mm

In the case of the mechanical properties, it is important to consider the forces applied, but it is the weakest point that determine the selection.

It is possible to change the geometry, but if you cannot What can we do?
Simplification: Train Wheel (Fast Example)
The Stiffness design is important to avoid excessive ELASTIC deflection.
The Stiffness design is important to avoid excessive ELASTIC deflection.
The Stiffness

\[ S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} \]
\[ \delta = \epsilon \cdot L \]

**EI** = Flexural rigidity
**I** = Second Moment of Inertia
**E** = Young’s Modulus
**δ** = Deflexion

Length (L): 300 mm
Thickness (h)= 1 mm
Width (b)= 25 mm

\[ I_{xx} = \frac{25 \cdot 1^3}{12} = 2.1 \, mm^4 \]
\[ I_{yy} = \frac{1 \cdot 25^3}{12} = 1300 \, mm^4 \]

**Problem:**

**δ??**
IF we consider that the beam is made of Stainless Steel (E = 200 GPa)

Which are the consequences if I want to use Polystyrene (E = 2 GPa)?
IF I can change the thickness and hold the same deflection.
The Stiffness

\[ S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} \]

- EI = Flexural rigidity
- I = Second Moment of inertia
- E = Young’s Modulus

Stainless Steel (E = 200 GPa; \( \rho = 7800 \text{ kg/m}^3 \))
Polystyrene (E = 2 GPa; \( \rho = 1040 \text{ kg/m}^3 \))

\[ I_{YY} = \frac{1 \cdot 25^3}{12} = 1300 \text{ mm}^4 \]
\[ I_{XX} = \frac{25 \cdot 1^3}{12} = 2.1 \text{ mm}^4 \]

\[ \delta = \frac{10 \cdot (0.25)^3}{3 \cdot (200 \cdot 10^9) \cdot (1300 \cdot 10^{-12})} = 0.02 \text{ mm} \] Steel

\[ \delta = \frac{FL^3}{C_1 E Y_{XX}} = 124 \text{ mm} \]

With \( \delta = 124 \text{ mm} \)
\[ I_{XX} = \frac{10 \cdot (0.25)^3}{3 \cdot (2 \cdot 10^9) \cdot (0.124)} = 210 \text{ mm}^4 \]

\[ h = \left( \frac{12I_{XX}}{w} \right)^{1/3} = \left( \frac{12 \cdot 210}{25} \right)^{1/3} = 4.6 \text{ mm} \] PS

When \( h(\text{Steel}) = 1 \text{ mm} \)
The Stiffness

\[ S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} \]

EI = Flexural rigidity
I = Second Moment of inertia
E = Young's Modulus
\( \delta = \) Deflexion

Length: 300 mm
Width: 25 mm

Stainless Steel (E = 200 GPa; \( \rho = 7800 \text{ kg/m}^3 \))
Polystyrene (E = 2 GPa; \( \rho = 1040 \text{ kg/m}^3 \))

Thickness = 1 mm
Thickness = 4.6 mm

About the weight?

\[ m_{SS} = 7800 \cdot 0.3 \cdot 0.025 \cdot 0.001 = 59 \text{ gr} \]
\[ m_{PS} = 1040 \cdot 0.3 \cdot 0.025 \cdot 0.046 = 36 \text{ gr} \]

BIGGER Section BUT LIGHTER

Depends on what you need and the conditions
The Materials Selection approach

**Case Study 1:** Find the Lightest STIFF Beam

- **Length:** 300 mm
- **Load:** 100 N
- **Cross-section Area:** 100 N

**Hypothesis:**
\[
\frac{F}{\delta} = S \geq S_{\text{min}}
\]

**Objective**
- Minimize the mass

**Constraints**
- Stiffness specified
- Length L
- Square shape

**Free Variables**
- Area (A) of the cross-section
- Choice of the material

**Equations**

- **EI** = Flexural rigidity
- I = Second Moment of inertia
- E = Young’s Modulus
- \( \delta = \text{Deflexion} \)

**Constants**
- \( C_1 = 3 \)

**Formulas**

- Mass:
  \[
  m = A \cdot L \cdot \rho
  \]

- Area:
  \[
  A = \frac{m}{L \cdot \rho}
  \]
The Materials Selection approach

Case Study 1: Find the Lightest STIFF Beam

\[ F \delta = C_1 E I \geq S_{\text{min}} \]

\[ A = \frac{m}{L \cdot \rho} \]

Since \( A = b^2 \)

\[ I = \frac{bh^3}{12} = \frac{A^2}{12} \]

The Area will be the Free Variable

\[ m \geq \left( \frac{12 \cdot S}{C_1 \cdot L} \right)^{1/2} \cdot L^3 \cdot \frac{\rho}{E^{1/2}} \]

Constraints

- Stiffness specified
- Length L
- Square shape
Case Study 1: Find the Lightest STIFF Beam

For instance \( \frac{E^{1/2}}{\rho} \)

\[ M = \frac{A}{B} \]

\[ \log(M) = \log(A) - \log(B) \]

\[ \log(A) = \log(B) + \log(M) \]

\[ \text{Selection Direction (+:)} \]

\[ \text{Slope} \]
Ashby Diagrams
Case Study 1: Find the Lightest STIFF Beam

The Material Index ($M$)

\[ m \downarrow \frac{E^{1/2}}{\rho} \uparrow \]

Stainless Steel
(E = 200 GPa; \( \rho = 7800 \text{ kg/m}^3 \))

Polystyrene
(E = 2 GPa; \( \rho = 1040 \text{ kg/m}^3 \))
Case Study 1: Find the Lightest STIFF Beam

F = 100 N  
δ = 0.34 mm  
S_{min} = 296 \cdot 10^3 \text{ N/m}

\[
m \geq \left( \frac{12 \cdot S}{C_1 \cdot L} \right)^{1/2} \cdot L^3 \cdot \frac{\rho}{E^{1/2}}
\]

\[
A = \frac{m}{L \cdot \rho}
\]

Length: 300 mm  
Thickness: 1 mm  
Width: 25 mm

Stainless Steel (E = 200 GPa; \(\rho = 7800 \text{ kg/m}^3\))  
Polystyrene (E = 2 GPa; \(\rho = 1040 \text{ kg/m}^3\))

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>A (mm²)</th>
<th>Width and Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steel</td>
<td>0.935</td>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.25</td>
<td>4000</td>
<td>63</td>
</tr>
</tbody>
</table>
Case Study 1: Find the Lightest STIFF Beam

To Minimize mass

Polystyrene (PS)

Polymers

Stainless steel

Metals and alloys

Young’s modulus (GPa)

Density (kg/m^3)

Slope

1

2
To Minimize Stiffness

- Depends on what you need
- It is not in flexion

Polystyrene (PS)
Metals and alloys
Polymers

Slope

Stainless steel
Ok, slow down..

**Case Study 2:**
*Find the Lightest STIFF Tie-Rod*

Tie-Rod =
\[ \text{TRACTION CONDITIONS} \]

**Objective**
- Minimize the mass

**Constraints**
- Stiffness specified
- Length \( L \)

**Free Variables**
- Area \( A \) of the cross-section
- Choice of the material

**DATA**
\( F = 1000 \text{ N} \)

**Dimensions:**
- Length: 300 mm
- Thickness = 1 mm
- Width = 25 mm

In Traction, the shape of the cross-section is not important:

\[
m = A \cdot L \cdot \rho \quad \Rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

From material:
\[
\frac{\sigma}{\varepsilon} = E
\]

From definition:
\[
\delta = \varepsilon \cdot L
\]

\[
F = \sigma \cdot A
\]

\[
\frac{F}{\delta} \geq S_{\text{min}} = S
\]
Case Study 2:
Find the Lightest STIFF Tie-Rod

\[
\frac{F}{\delta} \geq S_{\text{min}} = S
\]

\[
\frac{\sigma \cdot A}{\varepsilon \cdot L} \geq S_{\text{min}}
\]

\[
\frac{E \cdot A}{L} \geq S_{\text{min}}
\]

\[
m \geq S \cdot L^2 \cdot \frac{\rho}{E}
\]

\[
A = \frac{m}{L \cdot \rho}
\]

\[
m \geq (264,5 \cdot 10^6) \cdot (300 \cdot 10^{-3})^2 \cdot \frac{\rho}{E}
\]

\[
\geq \ldots \cdot \frac{\rho}{E}
\]

\begin{align*}
F &= 1000 \text{ N} \\
\delta &= 3,78 \cdot 10^{-3} \text{ mm} \\
S_{\text{min}} &= 264,5 \cdot 10^6 \text{ N/m} \\
\text{Dimensions:} & \\
\text{Length: } &= 300 \text{ mm} \\
\text{Thickness} &= 1 \text{ mm} \\
\text{Width} &= 25 \text{ mm}
\end{align*}
Case Study 2: Find the Lightest STIFF Tie-Rod
Case Study 2: Find the Lightest STIFF Tie-Rod

\[ \frac{E \cdot A}{L} \geq S_{\text{min}} \]

\[ A = \frac{m}{L \cdot \rho} \]

\[ m \geq S \cdot L^2 \cdot \frac{\rho}{E} \]

F = 1000 N
\( \delta = 3,78 \cdot 10^{-3} \text{ mm} \)
\( S_{\text{min}} = 264,5 \cdot 10^6 \text{ N/m} \)

**Dimensions:**
- Length: 300 mm
- Thickness = 1 mm
- Width = 25 mm

Stainless Steel (\( E = 200 \text{ GPa}; \rho = 7800 \text{ kg/m}^3 \))
Silicon carbide (\( E = 430 \text{ GPa}; \rho = 3150 \text{ kg/m}^3 \))
Al Alloys (\( E = 75 \text{ Gpa}; \rho = 2700 \text{ kg/m}^3 \))

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>A (mm(^2))</th>
<th>Width and Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon Carbide</td>
<td>0.174</td>
<td>179.8</td>
<td>13.4</td>
</tr>
<tr>
<td>Al Alloys</td>
<td>0.856</td>
<td>1050</td>
<td>32.4</td>
</tr>
</tbody>
</table>
Change of the section

**Beam: Square Section**

\[ b = h \]

**Panel:**

- **b fixed**
- **h free**

**Panel:**

- **h fixed**
- **b free**
Case Study 3: Find the Lightest STIFF Panel

Objective
• Minimize the mass

Constraints
• Stiffness specified
• Length L and b specified

Free Variables
• $h$ (thickness) of the cross-section
• Choice of the material

Hypothesis:
• $\frac{F}{\delta} \geq S = S_{\text{min}}$

Length: 300 mm
Width: 25 mm

\[
\frac{F}{\delta} \geq \frac{C_1 EI}{L^3} = S_{\text{min}}
\]

\[
m = A \cdot L \cdot \rho
\]

\[
A = \frac{m}{L \cdot \rho}
\]

$m =$ mass
$A =$ area of the section
$L =$ Length
$\rho =$ Density
Lightest Panel (Bending conditions)

**Case Study 3:**
*Find the Lightest STIFF Panel*

**Panel:**
- **b** fixed
- **h** free

Since $A = bh$

$$h = \frac{A}{b}$$

$$I = \frac{b \cdot (A/b)^3}{12} = \frac{A^3}{12 \cdot b^2}$$

$$S_{\text{min}} \leq \frac{C_1 E I}{L^3}$$

$$A = \frac{m}{L \cdot \rho}$$

The Area will be the Free Variable, but all the consequences of the selection are on the thickness

$$h = \frac{A}{b}$$

$$m \geq \left(\frac{12 \cdot S \cdot b^2}{C_1}\right)^{1/3} \cdot \frac{L^2 \cdot \rho}{E^{1/3}}$$

$$m \downarrow \frac{E^{1/3}}{\rho} \uparrow$$
Case Study 3: Find the Lightest STIFF Panel

\[ m = \frac{E^{1/3}}{\rho} \]
Bending conditions

Stainless Steel
(E = 200 GPa; ρ = 7800 kg/m³)
Polystyrene
(E = 2 GPa; ρ = 1040 kg/m³)
Bending conditions

F = 100 N  
δ = 0,34 mm  
S_{min} = 296 \cdot 10^3 \text{ N/m}

Stainless Steel (E = 200 \text{ GPa}; \rho = 7800 \text{ kg/m}^3)  
Polystyrene (E = 2 \text{ GPa}; \rho = 1040 \text{ kg/m}^3)

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>A (mm$^2$)</th>
<th>Thickness h (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steel</td>
<td>0,935</td>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1,25</td>
<td>4000</td>
<td>63</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>1,09</td>
<td>466</td>
<td>21,59</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0,67</td>
<td>2147</td>
<td>46,34</td>
</tr>
</tbody>
</table>

Beam  
Panel (b= 25 mm)
Stiffness Summary

\[
\begin{array}{ccc}
\frac{E}{\rho} & \frac{E^{1/2}}{\rho} & \frac{E^{1/3}}{\rho} \\
\end{array}
\]

To

\[
\begin{align*}
& m \quad \text{At fixed } S_{\text{min}} \\
& S_{\text{min}} \quad \text{and} \\
& \delta_{\text{max}} \quad \text{At fixed } m
\end{align*}
\]

Stiffness – Traction:

<table>
<thead>
<tr>
<th>Name</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon carbide</td>
<td>0.136</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>0.0984</td>
</tr>
<tr>
<td>Alumina</td>
<td>0.094</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0657</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0634</td>
</tr>
<tr>
<td>Tungsten carbides</td>
<td>0.0425</td>
</tr>
<tr>
<td>Silica glass</td>
<td>0.0323</td>
</tr>
<tr>
<td>Soda-lime glass</td>
<td>0.0284</td>
</tr>
<tr>
<td>Borosilicate glass</td>
<td>0.0278</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>0.0277</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.025</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.0158</td>
</tr>
</tbody>
</table>

Stiffness – Bending:

<table>
<thead>
<tr>
<th>Name</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon carbide</td>
<td>0.00657</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.00651</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.00601</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>0.00546</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.00522</td>
</tr>
<tr>
<td>Alumina</td>
<td>0.00492</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.00478</td>
</tr>
<tr>
<td>Rigid Polymer Foam (LD)</td>
<td>0.00413</td>
</tr>
<tr>
<td>Silica glass</td>
<td>0.00384</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.00362</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.00354</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.00314</td>
</tr>
</tbody>
</table>

Stiffness – Bending:

<table>
<thead>
<tr>
<th>Name</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Polymer Foam (LD)</td>
<td>0.00697</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.00442</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.00373</td>
</tr>
<tr>
<td>Flexible Polymer Foam (VLD)</td>
<td>0.00335</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.00321</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.00301</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.00269</td>
</tr>
<tr>
<td>Rigid Polymer Foam (HD)</td>
<td>0.00239</td>
</tr>
<tr>
<td>Flexible Polymer Foam (LD)</td>
<td>0.00233</td>
</tr>
<tr>
<td>Flexible Polymer Foam (MD)</td>
<td>0.00212</td>
</tr>
<tr>
<td>Cork</td>
<td>0.00173</td>
</tr>
</tbody>
</table>
Case Study 4: Materials for Oars
Case Study 4: Materials for Oars

Objective
- Minimize the mass

Constraints
- Stiffness specified
- Length L
- Circular shape (beam)

Free Variables
- Area (A) of the cross-section
- Choice of the material

L (Outboard) = 2 m
Case Study 4: Materials for Light Oars

We assume solid section

\[ A = \pi \cdot r^2 \]

\[ I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi} \]

\[
\begin{cases}
\frac{F}{\delta} \geq S_{\text{min}} = \frac{C_1EI}{L^3} \\
A = \frac{m}{L \cdot \rho}
\end{cases}
\]

\[
\begin{cases}
A = \frac{m}{L \cdot \rho} \\
m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}}
\end{cases}
\]
**Case Study 4: Materials for Light Oars**

The diagram shows a plot comparing the Young's modulus (GPa) and density (kg/m³) of various materials. The materials considered are:

- **Silicon carbide**
- **CFRP, epoxy matrix (isotropic)**
- **Bamboo**
- **Paper and cardboard**
- **Rigid Polymer Foam (LD)**
- **Rigid Polymer Foam (MD)**

The legend indicates the stage 1 index for each material:

- Silicon carbide: 0.00657
- CFRP, epoxy matrix (isotropic): 0.00651
- Bamboo: 0.00601
- Paper and cardboard: 0.00546
- Aluminum nitride: 0.00522
- Silicon: 0.00492
- Wood, typical along grain: 0.00478
- Rigid Polymer Foam (LD): 0.00413
- Paper and cardboard: 0.00354
- Rigid Polymer Foam (MD): 0.00314
Case Study 4: Materials for Light and Slender Oars

We assume solid section

\[ A = \pi \cdot r^2 \quad I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi} \]

\[
\begin{cases}
  \frac{F}{\delta} \geq S_{\text{min}} = \frac{C_1 EI}{L^3} \\
  I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi}
\end{cases}
\]

\[ A \leq \left( \frac{4 \cdot \pi \cdot S \cdot L^3}{3} \right)^{1/2} \cdot \frac{1}{E^{1/2}} \]

Place LIMITS to a single Property Evaluating the Properties Chart

10 GPa < E < 200 GPa
Case Study 4: Materials for Light and Slender Oars

CFRP - best material with more control of the properties
Bamboo – Traditional material for oars for canoes
Woods – Traditional, but with natural variabilities
Ceramics – Low toughness and high cost
Case Study 4: Materials for Light and Slender Oars

Solid \( I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi} \)

Tube \( I = \pi r^3 t \)

\[ S \geq \frac{3 \cdot m^2}{4 \cdot \pi \cdot L^5} \cdot \frac{E}{\rho^2} \]

\( m \quad \text{At fixed } S_{\text{min}} \)

\( S_{\text{min}} \quad \uparrow \) \quad \text{At fixed } m \quad \delta_{\text{max}} \downarrow

1.58 kg
Probably Tube shape
Assume 2.5 kg for a Solid Oar (exaggerated)

CFRP (\( E = 110 \text{ GPa}; \rho = 1550 \text{ kg/m}^3 \))
Bamboo (\( E = 17.5 \text{ GPa}; \rho = 700 \text{ kg/m}^3 \))

CFRP good for Competition Oar
**Case Study 5:**
**Materials for CHEAP and Slender Oars**

<table>
<thead>
<tr>
<th>Objective</th>
<th>• Minimize the cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>• Stiffness specified</td>
</tr>
<tr>
<td></td>
<td>• Length L</td>
</tr>
<tr>
<td></td>
<td>• Circular shape (beam)</td>
</tr>
<tr>
<td>Free Variables</td>
<td>• Area (A) of the cross-section</td>
</tr>
<tr>
<td></td>
<td>• Choice of the material</td>
</tr>
</tbody>
</table>

![Diagram of oar parts]

$L \text{ (Outboard)} = 2 \text{ m}$
**Case Study 5:**  
*Materials for CHEAP and Slender Oars*

\[
m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}}
\]

\[
C = m \cdot C_m \quad \rightarrow \quad m = \frac{C}{C_m}
\]

**Cost**  
\(C\)  
\(C_m\) Cost per unit of mass

Better to consider cost always as a function of mass

\[
C \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho \cdot C_m}{E^{1/2}}
\]

\[
\frac{E^{1/2}}{\rho \cdot C_m}
\]
Case Study 5:
Materials for CHEAP and Slender Oars

Bamboo – Traditional material for oars for canoes
Woods – Traditional, but with natural variabilities
Stone and Concrete – Low toughness and difficult to manufacture

\[
C = \frac{E^{1/2}}{\rho \cdot C_m}
\]

10 GPa < E < 200 GPa
Case Study 5: Materials for CHEAP and Slender Oars

\[
m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}}
\]

\[
C = m \cdot C_m \quad \Rightarrow \quad m = \frac{C}{C_m}
\]

Cost

\[C_m\] Cost per unit of mass

\[\downarrow\]

Better to consider cost always as a function of mass

Woods good for Commercial Oar
The Strength design is important to avoid plastic collapse... or maybe not
The Strength design is important to avoid plastic collapse.
Lightest Tie-Rod (Traction conditions)

**Case Study 6:** Find the Lightest STRONG Tie-Rod

**Objective**
- Minimize the mass

**Constraints**
- Support tensile load $F$ without yielding
- Length $L$

**Free Variables**
- Area $(A)$ of the cross-section
- Choice of the material

**DATA**
- $F = 1000$ N
- **Dimensions:**
  - Length: 300 mm
  - Thickness = 1 mm
  - Width = 25 mm

In Traction, the shape of the cross-section is not important

\[
m = A \cdot L \cdot \rho
\]

\[
A = \frac{m}{L \cdot \rho}
\]

From material:
\[
\frac{F}{A} \leq \sigma_y
\]

\[
m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}
\]
Ashby Diagrams

Strength – Density

- Metals and polymers: yield strength, $\sigma_y$
- Ceramics, glasses: modulus of rupture, MOR
- Elastomers: tensile tear strength, $\sigma_t$
- Composites: tensile failure, $\sigma_f$

Guide lines for minimum mass design
Case Study 6: Find the Lightest STRONG Tie-Rod

![Graph showing materials selection criteria]

- CFRP, epoxy matrix (isotropic)
- Low alloy steel
- Silicon carbide
- Cast iron, ductile (nodular)
- Aluminum alloys
- Magnesium alloys
- Wood, typical along grain

Criteria:
- \( m \downarrow \)
- \( \frac{\sigma_y}{\rho} \uparrow \)
Lightest Tie-Rod (Traction conditions)

Case Study 6: Find the Lightest STRONG Tie-Rod

\[ m \geq F \cdot L \cdot \frac{\rho}{\sigma_y} \]

It is possible to do as before, but let's calculate the maximum F on the precedent Tie-Rod

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>Width and Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al Alloys</td>
<td>1.25</td>
<td>63</td>
</tr>
</tbody>
</table>

Stainless Steel (\(\sigma_y = 600\) MPa; \(\rho = 7800\) kg/m\(^3\))
Wood (\(\sigma_y = 50\) MPa; \(\rho = 700\) kg/m\(^3\))
Al Alloys (\(\sigma_y = 270\) Mpa; \(\rho = 75\) kg/m\(^3\))

\[ F \leq \frac{m \cdot \sigma_y}{L \cdot \rho} = 416 \text{ kN} \]

Elastic Throughout

Plastic deformation/ Collapse
Lightest Beam (Bending conditions)

Case Study 7: Find the Lightest STRONG Beam

Objective
• Minimize the mass

Constraints
• Stiffness specified
• Length L
• Square shape

Free Variables
• Area (A) of the cross-section
• Choice of the material

M = Moment
σ = Stress

\[
\sigma_{\text{max}} = \frac{M_f \cdot y_{\text{max}}}{I} \leq \sigma_f
\]

\[
m = A \cdot L \cdot \rho \quad \Rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

\[
L = \frac{bh^3}{12} \quad \Rightarrow \quad A^{3/2} = \frac{6b}{h^3}
\]

\[
y_{\max} = \text{max distance from the neutral axis} \rightarrow \sigma_{\text{max}}
\]

C₁ = 3

Length: 300 mm

Hypothesis:
• \( y_{\text{max}} = h/2 \)
• \( \sigma_{\text{max}} \geq \sigma \)

Beam: Square Section

b = h

Since \( A = b^2 \)
Lightest Beam (Bending conditions)

Case Study 7: Find the Lightest STRONG Beam

\[ \sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot \frac{h}{2}}{bh^3/12} = \frac{M_f}{I'} \leq \sigma_f \]

\[ m = A \cdot L \cdot \rho \quad \Rightarrow \quad A = \frac{m}{L \cdot \rho} \]

\[ I' = \frac{bh^2}{6} \implies \frac{A^{3/2}}{6} \]

\[ \sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}} \]

\[ m \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho}{\sigma_f^{2/3}} \]
Case Study 7: Find the Lightest STRONG Beam

\[ m = \frac{\sigma_y^{2/3}}{\rho} \]
Lightest Beam (Bending conditions)

Case Study 7: Find the Lightest STRONG Beam

\[
I' = \frac{bh^2}{6} \gg \frac{A^{3/2}}{6}
\]

\[
\sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot h}{bh^3} = \frac{M_f}{I'} \leq \sigma_f
\]

\[
m = A \cdot L \cdot \rho \quad \Rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

- It is always better to choose a shape that uses less material to provide the same strength TO SUPPORT BENDING

\[
\sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}}
\]

\[
m \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho}{\sigma_f^{2/3}}
\]
Case Study 8: Find the Lightest STRONG Panel

Objective

• Minimize the mass

Constraints

• Stiffness specified
• Length L and b specified

Free Variables

• h (thickness) of the cross-section
• Choice of the material

Hypothesis:

• $y_{\text{max}} = h/2$
• $\sigma_{\text{max}} \geq \sigma$

\[
\sigma_{\text{max}} = \frac{M_f \cdot y_{\text{max}}}{I} \leq \sigma_f
\]

\[
m = A \cdot L \cdot \rho \quad \rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

\[
I' = \frac{b h^2}{6} \gg \frac{A^2}{b \cdot 6}
\]

Since $A = bh$

\[
h = \frac{A}{b}
\]
Lightest Panel (Bending conditions)

Case Study 8:
Find the Lightest STRONG Panel

\[
l' = \frac{bh^2}{6} \gg \frac{A^2}{b \cdot 6}
\]

\[
\sigma_{max} = \frac{M_f \cdot \gamma_{max}}{I} = \frac{M_f \cdot b \cdot 6}{A^2} = \frac{M_f}{I'} \leq \sigma_f
\]

\[
m = A \cdot L \cdot \rho \quad \rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

\[
\sigma_f \geq \frac{M_f \cdot b \cdot 6}{A^2} = \frac{M_f \cdot 6 \cdot b \cdot L^2 \cdot \rho^2}{m^2}
\]

\[
m \geq (M_f \cdot 6 \cdot b)^{1/2} \cdot L \cdot \frac{\rho}{\sigma_f^{1/2}}
\]
Case Study 8: Find the Lightest STRONG Panel

- CFRP, epoxy matrix (isotropic)
- Magnesium alloys
- Wood, typical along grain
- Rigid Polymer Foam (HD)
- Rigid Polymer Foam (LD)

\[ m \downarrow \frac{\sigma_y^{1/2}}{\rho} \uparrow \]

- Name
  - CFRP, epoxy matrix (isotropic)
  - Rigid Polymer Foam (LD)
  - Wood, typical along grain
  - Rigid Polymer Foam (HD)
  - Bamboo
  - Paper and cardboard
  - Magnesium alloys
  - Rigid Polymer Foam (MD)
  - Flexible Polymer Foam (VLD)
  - Flexible Polymer Foam (LD)

Stage 1: Index
- 0.0178
- 0.0168
- 0.00977
- 0.00959
- 0.00904
- 0.00787
- 0.0074
- 0.00702
- 0.00623
- 0.0054
Lightest Panel (Bending conditions)

Case Study 8: Find the Lightest STRONG Panel

\[
\sigma_{\text{max}} = \frac{M_f \cdot y_{\text{max}}}{I} = \frac{M_f \cdot \frac{h}{2}}{bh^3} = \frac{M_f}{I'} \leq \sigma_f
\]

\[
m = A \cdot L \cdot \rho \quad \Rightarrow \quad A = \frac{m}{L \cdot \rho}
\]

- **It is always better to choose a shape that uses less material to provide the same strength TO SUPPORT BENDING**

\[
I' = \frac{bh^2}{6} \gg \frac{A^{3/2}}{6}
\]

\[
\sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}}
\]

\[
m \geq (M_f \cdot 6 \cdot b)^{1/2} \cdot L \cdot \frac{\rho}{\sigma_f^{1/2}}
\]

\[
m \downarrow \quad \frac{\sigma_y^{1/2}}{\rho} \uparrow
\]
Summary (to minimize the mass)

### Materials Selection

**Stiffness – Traction:**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon carbide</td>
<td>0.136</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>0.0984</td>
</tr>
<tr>
<td>Alumina</td>
<td>0.094</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0657</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0634</td>
</tr>
<tr>
<td>Tungsten carbides</td>
<td>0.0425</td>
</tr>
<tr>
<td>Silica glass</td>
<td>0.0323</td>
</tr>
<tr>
<td>Soda-lime glass</td>
<td>0.0284</td>
</tr>
<tr>
<td>Borosilicate glass</td>
<td>0.0278</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>0.0277</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.025</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.0158</td>
</tr>
</tbody>
</table>

**Stiffness – Bending (Beam):**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon carbide</td>
<td>0.0657</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0651</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.0601</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>0.0646</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.0532</td>
</tr>
<tr>
<td>Alumina</td>
<td>0.0492</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.0478</td>
</tr>
<tr>
<td>Rigid Polymer Foam (LD)</td>
<td>0.0413</td>
</tr>
<tr>
<td>Silica glass</td>
<td>0.0384</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.0362</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.0354</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.0314</td>
</tr>
</tbody>
</table>

**Stiffness – Bending (Panel):**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Polymer Foam (LD)</td>
<td>0.0697</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.0442</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.0373</td>
</tr>
<tr>
<td>Flexible Polymer Foam (MD)</td>
<td>0.0335</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.0321</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0301</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.0269</td>
</tr>
<tr>
<td>Rigid Polymer Foam (HD)</td>
<td>0.0239</td>
</tr>
<tr>
<td>Flexible Polymer Foam (MD)</td>
<td>0.0212</td>
</tr>
<tr>
<td>Cork</td>
<td>0.0173</td>
</tr>
</tbody>
</table>

**Strength – Traction:**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stage 1: Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.491</td>
</tr>
<tr>
<td>Silicon carbide</td>
<td>0.157</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>0.121</td>
</tr>
<tr>
<td>Alumina</td>
<td>0.117</td>
</tr>
<tr>
<td>Low alloy steel</td>
<td>0.0987</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>0.0983</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.0908</td>
</tr>
<tr>
<td>High carbon steel</td>
<td>0.0866</td>
</tr>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0793</td>
</tr>
<tr>
<td>Medium carbon steel</td>
<td>0.0667</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.0661</td>
</tr>
<tr>
<td>Cast iron, ductile (nodular)</td>
<td>0.0577</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>0.0526</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>0.0455</td>
</tr>
<tr>
<td>Nickel alloys</td>
<td>0.0312</td>
</tr>
</tbody>
</table>

**Strength – Bending (Panel):**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>CFRP, epoxy matrix (isotropic)</td>
<td>0.0178</td>
</tr>
<tr>
<td>Rigid Polymer Foam (LD)</td>
<td>0.0168</td>
</tr>
<tr>
<td>Wood, typical along grain</td>
<td>0.00977</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.00959</td>
</tr>
<tr>
<td>Bamboo</td>
<td>0.00904</td>
</tr>
<tr>
<td>Flexible Polymer Foam (VLD)</td>
<td>0.00787</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.0074</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.00702</td>
</tr>
<tr>
<td>Rigid Polymer Foam (HD)</td>
<td>0.00623</td>
</tr>
<tr>
<td>Flexible Polymer Foam (LD)</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

**Strength – Bending:**

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<tr>
<td>Wood, typical along grain</td>
<td>0.00977</td>
</tr>
<tr>
<td>Rigid Polymer Foam (MD)</td>
<td>0.00959</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>0.00904</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.00787</td>
</tr>
<tr>
<td>Paper and cardboard</td>
<td>0.0074</td>
</tr>
<tr>
<td>Magnesium alloys</td>
<td>0.00702</td>
</tr>
<tr>
<td>Rigid Polymer Foam (HD)</td>
<td>0.00623</td>
</tr>
<tr>
<td>Flexible Polymer Foam (LD)</td>
<td>0.0054</td>
</tr>
</tbody>
</table>
Case Study 9: Materials for Constructions

Some data: Nowadays, half the expense of building a house is the cost of the materials
Family house: 200 tons
Large apartment block: 20,000 tons
Case Study 9: Materials for Constructions

Mr. Pincopallo asks a new cover

Understand the problem and translate it in selection criteria, thus properties,

A cover has 3 BROAD ROLES:

Cladding: Protection from the environment

The frames: Mechanical support

Internal surfacing: Control heat, light and sound
**Case Study 9: Materials for Constructions (Structural Frame)**

<table>
<thead>
<tr>
<th>Objective</th>
<th>• Minimize the cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>• Length L specified</td>
</tr>
<tr>
<td></td>
<td>• Stiffness: must not deflect too much under loads</td>
</tr>
<tr>
<td></td>
<td>• Strength: must not fall under design loads</td>
</tr>
<tr>
<td>Free Variables</td>
<td>• Area (A) of the cross-section</td>
</tr>
<tr>
<td></td>
<td>• Choice of the material</td>
</tr>
</tbody>
</table>

Hypothesis:
- \( \frac{F}{\delta} \geq S_{\text{min}} = S \)

**Floor joints are beams, loaded in bending.**

\[
\begin{align*}
\frac{F}{\delta} & \geq S_{\text{min}} = \frac{C_1 EI}{L^3} \\
m & = A \cdot L \cdot \rho \quad \rightarrow \quad A = \frac{m}{L \cdot \rho} \\
C & = m \cdot C_m \quad \rightarrow \quad m = \frac{C}{C_m} \\
C & \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho \cdot C_m}{E^{1/2}}
\end{align*}
\]
Case Study 9: Materials for Constructions (Structural Frame)

Objective
- Minimize the cost

Constraints
- Length L specified
- Stiffness: must not deflect too much under loads
- Strength: must not fall under design loads

Free Variables
- Area (A) of the cross-section
- Choice of the material

Floor joints are beams, loaded in bending.

\[ I = \frac{bh^3}{12} \gg \frac{A^{3/2}}{6} \]

\[ \sigma_{\text{max}} = \frac{M_f \cdot y_{\text{max}}}{I} \leq \sigma_f \]

\[ m = A \cdot L \cdot \rho \quad \rightarrow \quad A = \frac{m}{L \cdot \rho} \]

\[ C = m \cdot C_m \quad \rightarrow \quad m = \frac{C}{C_m} \]

\[ C \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho \cdot C_m}{\sigma_f^{2/3}} \]
Case Study 9: Materials for Constructions

ATTENTION!!!
Selection with the cost/kg and with the cost/m³ is DIFFERENT
**Case Study 9: Materials for Constructions**

- Low carbon steel
- Cast iron, ductile (nodular)
- Aluminum alloys
- Stone
- Concrete
- Wood, typical along grain
- Paper and cardboard

Mathematical expression:

\[
C \downarrow \frac{E^{1/2}}{\rho \cdot C_m} \uparrow
\]
Case Study 9: Materials for Constructions

\[ C \left( \frac{\sigma_f^{2/3}}{\rho \cdot C_m} \right) \]

Diagram showing materials selection for constructions with different properties such as tensile strength, density, and price. The diagram categorizes materials into different zones for optimization based on the given formula.
Case Study: Materials for Springs
### Case Study 10: Materials for Small Springs

**Objective**  
• Maximize stored elastic energy

**Constraints**  
• No failure \( \sigma < \sigma_f \) throughout the spring

**Free Variables**  
• Choice of the material

\[
\begin{align*}
\text{Condition of elasticity} & = \begin{cases} 
\sigma_y \geq \sigma \\
\sigma = E \cdot \varepsilon \quad \Rightarrow \quad \varepsilon = \frac{\sigma}{E}
\end{cases} \\
SMALL?? \rightarrow V \text{ FREE VARIABLE}!! \\
dV = dx dy dz
\end{align*}
\]

[Solid Mechanics Part I Kelly]
Case Study 10: Materials for Small Springs

\[
\begin{align*}
\sigma_y & \geq \sigma \\
\varepsilon &= \frac{\sigma}{E}
\end{align*}
\]

\[dV = dx dy dz\]

\[
\begin{align*}
m &= V \cdot \rho \\
W_{el} &= \frac{1}{2} \int \sigma \cdot \varepsilon \, dV = \frac{1}{2} \sigma \cdot \varepsilon \cdot V
\end{align*}
\]

Total strain energy in the piece considered

\[U = \frac{\left(\sigma_{xx} dy dz\right)^2 dx}{2Edy dz}\]

\[W_{el} = \frac{\sigma_y^2}{2E} = M_1\]

Total strain energy PER UNIT OF VOLUME

[Solid Mechanics Part I Kelly]
Case Study 10: Materials for Small Springs

- **CFRP** = Comparable in performance with steel; expensive [TRUCK SPRINGS]
- **STEEL** = The traditional choice: easily formed and heat treated
- **TITANIUM** = Expensive, corrosion resistant

- **RUBBERS** = have Excellent $M_1$ but low tensile strength high loss factor
- **NYLON** = Inexpensive and easily shaped, but high loss factor [CHILDREN’S TOYS]
Case Study
Materials for Springs

\[ W_{el} = \frac{\sigma_y^2}{2E} \]

Valid for axial springs
Because much of the material is not fully loaded

PAY ATTENTION

\[ W_{el} = \frac{\sigma_y^2}{3E} \]

For torsion springs (less efficient)
Case Study
Materials for Springs

\[ W_{el} = \frac{\sigma_y^2}{4E} \]

For leaf springs (less efficient)
Case Study 9: Materials for Light Springs

\[
\begin{align*}
\sigma_y & \geq \sigma \\
\varepsilon & = \frac{\sigma}{E}
\end{align*}
\]

\[dV = dx dy dz\]

\[m = V \cdot \rho\]

\[
W_{el} = \frac{1}{2} \int \sigma \cdot \varepsilon \, dV = \frac{1}{2} \sigma \cdot \varepsilon \cdot V
\]

Total strain energy in the piece considered

Strain energy density

\[\sigma \]

\[\varepsilon \]

\[u\]

\[LIGHT??? \rightarrow \frac{V}{\rho} \text{ FREE VARIABLE!!}\]

\[
\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2
\]

Total strain energy

PER UNIT OF MASS

[Solid Mechanics Part I Kelly]
Case Study 11:
Materials for Light Springs

- **CFRP** = Comparable in performance with steel; expensive [TRUCK SPRINGS]
- **RUBBERS** = 20 times better than Steel; but low tensile strength high loss factor
- **NYLON** = Inexpensive and easily shaped, but high loss factor [CHILDREN’S TOYS]
Case Study 12: Materials for Car Body

Some context → Car Evolution

1932 Ford Model B

1934 Bonnie and Clyde car
Case Study 12: Materials for Car Body

Some context → Car Evolution

1932 Ford Model B

1970 Buick GSX

2010 Ferrari 458 Italia

Km/h

80 km/h

184 km/h

325 km/h
Case Study 12: Materials for Car Body

Deformation? → ENERGY CONSUMPTION

At first, automotive industry move to too deformable cars and then move to have a mix FOR PEOPLE SAFETY.

Sometimes exaggerate
## Case Study 12: Materials for Car Body (Car Hood or Car Door)

<table>
<thead>
<tr>
<th>Objective</th>
<th>• Maximize plastic deformation at high load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>• Geometry</td>
</tr>
<tr>
<td></td>
<td>• High $\sigma_y$</td>
</tr>
<tr>
<td></td>
<td>• Division for price</td>
</tr>
<tr>
<td></td>
<td>• Consider manufacture</td>
</tr>
<tr>
<td>Free Variables</td>
<td>• Choice of the material</td>
</tr>
</tbody>
</table>

**Objective**

- Maximize plastic deformation at high load

**Constraints**

- Geometry
- High $\sigma_y$
- Division for price
- Consider manufacture

**Free Variables**

- Choice of the material

**Total strain energy PER UNIT OF MASS**

$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

**LIGHT?? → m**

**Diagram**

- Car body with various parts labeled:
  - Trunk
  - Tail light
  - Back fender
  - Quarter window
  - Roof post
  - Roof
  - Sunroof
  - Windshield
  - Outside mirror
  - Windshield wiper
  - Hood
  - Grill
  - Headlight
  - Bumper
  - License plate
  - Indicator light
  - Shield
  - Front wheel
  - Hub cap
  - Door post
  - Door
  - Door handle
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[ W_{el} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2 \]

Total strain energy PER UNIT OF MASS

**Objective**
- Maximize plastic deformation at high load

**Constraints**
- Geometry
- High \( \sigma_y \)
- Division for price
- Consider manufacture

**Free Variables**
- Choice of the material

**Steps:**
- **Stiffness selection (Take off flexible materials)**
- **Yield strength selection to minimize the costs (Automotive)**
- **Minimum Yield Strength**
- **Maximization of stored energy**
Case Study 12: Materials for Car Body (Car Hood or Car Door)
Case Study 12: Materials for Car Body (Car Hood or Car Door)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

+ minimum $\sigma_y$ (200 MPa)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[ W_{el} = \frac{\sigma_f^2}{2E} = M_1 \]

Total strain energy PER UNIT OF VOLUME
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[
\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2
\]

Total strain energy PER UNIT OF MASS
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[
\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2
\]

Total strain energy PER UNIT OF MASS

Lamborghini Huracan 2015
Manufacturing Consideration

Changing the carbon fiber manufacturing process

Lamborghini’s innovation is a product and a process called Forged Composite. This material starts off as a sheet of uncured plastic that is mixed with short lengths of randomly placed carbon fiber strands. Unlike traditional pre-preg carbon fiber cloth, you don’t have to carefully cut this material and lay it out precisely in a mold. You just have to cut off the right mass and put the chunk into a hot press mold. You squeeze it, heat it and you’re done. The part that comes out of the mold is as light (or lighter) and as stiff (or stiffer) than a conventionally laid-up carbon fiber part, and you can produce it in minutes rather than hours.

You can now treat carbon fiber the way the automobile industry has treated steel, aluminum, and unreinforced plastic for decades.

This changes the rules of manufacturing because you can now treat carbon fiber the way the automobile industry (and every other manufacturing industry) has treated steel, aluminum, and unreinforced plastic for decades: You just stamp out the parts you need. As automakers look to the future of increased CAFE standards and lighter-weight vehicles, making parts out of carbon fiber without the extra labor expense is a killer app.

“By continuing to develop our patented forged composite materials, we are able to create a product that can enhance Lamborghini super sports cars in both their performance and their appearance,” said Maurizio Reggiani, Director of R&D for Lamborghini. “The ability to leverage this kind of lightweight material gives Lamborghini an advantage that will benefit our cars – as well as production process – in the future.”

[www.digitaltrends.com/cars/lamborghini-forged-carbon-fiber-manufacturing-process]
Case Study 12: Materials for Car Body (Car Hood or Car Door)

Vacuum and pressure bag molding

Not adapt for high productions

No way for a commercial car

Cost model and defaults

Relative cost index (per unit)

Parameters:
- Material Cost = 7.33EUR/kg
- Component Mass = 1kg
- Batch Size = 1e3
- Overhead Rate = 137EUR/hr
- Discount Rate = 5%
- Capital Write-off Time = 5yrs
- Load Factor = 0.5

<table>
<thead>
<tr>
<th>Batch Size (units)</th>
<th>Relative cost index (per unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>1e2</td>
<td>2500</td>
</tr>
<tr>
<td>1e3</td>
<td>1300</td>
</tr>
<tr>
<td>1e4</td>
<td>500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capital cost</th>
<th>Material utilization fraction</th>
<th>Production rate (units)</th>
<th>Tooling cost</th>
<th>Tool life (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.01e4</td>
<td>0.85</td>
<td>0.05</td>
<td>752</td>
<td>100</td>
</tr>
</tbody>
</table>
Case Study 12: Materials for Car Body (Car Hood or Car Door)

And which Metal?

\[
\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2
\]

Total strain energy PER UNIT OF MASS
Case Study 12: Materials for Car Body (Car Hood or Car Door)

Stamping

Cost model and defaults

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Material Cost</td>
<td>7.33 EUR/kg</td>
</tr>
<tr>
<td>Component Mass</td>
<td>1 kg</td>
</tr>
<tr>
<td>Batch Size</td>
<td>1e3</td>
</tr>
<tr>
<td>Overhead Rate</td>
<td>137 EUR/hr</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>5%</td>
</tr>
<tr>
<td>Capital Write-off Time</td>
<td>5 yrs</td>
</tr>
<tr>
<td>Load Factor</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Relative cost index (per unit)

Adapt for high productions

<table>
<thead>
<tr>
<th>Batch Size</th>
<th>Relative cost index (per unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>10000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Capital cost

Material utilization fraction

Production rate (units)

Tooling cost

Tool life (units)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital cost</td>
<td>7.52e3 - 7.52e4 EUR</td>
</tr>
<tr>
<td>Material utilization fraction</td>
<td>0.7 - 0.8</td>
</tr>
<tr>
<td>Production rate (units)</td>
<td>200 - 5e3 /hr</td>
</tr>
<tr>
<td>Tooling cost</td>
<td>150 - 1.5e4 EUR</td>
</tr>
<tr>
<td>Tool life (units)</td>
<td>1e4 - 1e6</td>
</tr>
</tbody>
</table>
Case Study 12: Materials for Car Body (Car Hood or Car Door)

*PROCESSABILITY*

*Metals easy to stamp*

![Graph showing processability of different materials for car body parts.](image-url)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[ W_{el} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2 \]

Total strain energy \(\text{PER UNIT OF MASS}\)

+ Minimum Formability (4)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

*PROCESSABILITY*

*Metals easy to stamp*

![Diagram showing processability of materials for car body](image)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

\[ \frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2 \]

Total strain energy PER UNIT OF MASS

+ Minimum Formability (4)
+ Minimum Brazability (3)
Case Study 12: Materials for Car Body (Car Hood or Car Door)

**BMW M3 – Low alloy steel**

**Audi A8 – Al-alloys**

**Deeper selection?**

LEVEL 3

Materials Selection Steps

- Decide Design Requirements
  - Get rid of Candidates that don’t fit constraints e.g. max service temperature isn’t high enough
  - Optimize on Objectives e.g. low mass, low cost, high strength

- Scrutinize candidate shortlist – do I have valid properties

- Materials Selection is about trade-offs, not one right answer
- Environmental legislation, processability and the security of the supply chain are important factors, along side mechanical and thermal performance

Optional:
- Decide on strategy to fill knowledge gaps